NUMERICAL INVESTIGATION OF REPEATED MACH REFLECTION AND AMPLIFICATION OF SHOCK WAVES CONVERGING TO THE VERTEX OF A CONE

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Results are given for numerical calculations of two-dimensional gasdynamic accumulation for the convergence of a plane shock wave to the vertex of a cone and for the compression of a gas in the cone of a piston.

It is of interest to solve problems about the convergence of a plane shock wave to the vertex of a cone, together with technical applications, connected with the more general problem of two-dimensional gasdynamic accumulation. The number of studies devoted to the investigation of two-dimensional, cumulative problems is comparatively small. Among them, we can note the paper [1], where a qualitative analysis is made for the picture of repeated Mach reflection of shock waves having frequency approaching infinity, with the wave front approaching the vertex of a wedge-shaped cavity. In that paper, experimental data is given that shows sizable growth in the observed intensity of the visible radiation ($\sim10^3$) and density ($\sim10^2$ times) with respect to their initial values behind the front of an incoming shock wave in air. In [2, 3] diffraction and amplification were considered for a plane shock wave on the walls and axis of symmetry for convergence of the wave to the vertex of the cone. Using the Chester—Chiznell—Whitham (CCW) method [4] we obtain a solution predicting amplification of the shock-wave intensity for asymptotic approach to the vertex of a wedge-shaped cavity or cone according to a law close to a power dependence of the self-similar Guderley—Landau—Stanyukovich solution [5, 6] for a cylindrical or spherical shock wave. The theoretical values of the shock-wave velocity on the axis of symmetry of the cone calculated by the CCW method agree well with the data of experimental measurements in argon [2] and air [3]. In a number of studies, experiments are described in which gaseous deuterium or a deuterium—tritium mixture was compressed in the conical cavity of a shell accelerated by x-rays obtained in the conversion of REP energy [7], laser radiation [8, 9], and using explosives [10, 11]. A theoretical analysis of such compression was usually limited to a one-dimensional approximation, in which consideration of the cone was replaced by a spherical interaction energy that decreased by a factor of $4\pi/\Omega$. Calculations [7, 9, 11] showed that compression and heating of deuterium occur mainly as a result of repeated reflection of a shock wave at the vertex of a cone and from the inside of the shell, which in this case is damped and imparts its energy to the gas. At the same time, in laser experiments [8, 9, 12], compression can evidently be distinguished from a quasi-spherical shell since in certain cases the shell always has a small curvature. Deformation of the walls with motion of the compressible gas of the piston was taken into account in two-dimensional calculations with the use of slip grids [13]. Here, a comparison of average calculation parameters according to a two-dimensional method with similar parameters, obtained in an approximate model, showed good agreement. Taking account of what has been said, it is advisable to calculate the compression of deuterium in two-dimensional geometry with a plane piston. The aim of the present work is the numerical investigation of the problem of a plane shock wave, converging to a cone vertex, and either generated in a shock tube or arising as a result of the acceleration of a plane shell accelerated by a laser pulse.

1. In a first approximation we take the gasdynamic formulation of the problem neglecting dissipative processes of viscosity, thermal conductivity, transport of energy by radiation, compressibility and deformations of the walls of the cone and the shell. The system of partial differential equations in divergent form is written as

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TABLE 1. Maximum Values of Gasdynamic Quantities in the Vertex of a Cone for Various Piston Acceleration Regimes

Variant	t_m, nsec	E_m, J	P_m, GPa	ρ_m, mg/cm³	T_m, 10^6 K	N
1	81,1	0,84	42	4,65	1,09	None
2	64,6	4,80	190	4,50	5,09	$2\cdot10^2$
3	64,3	5,02	204	4,60	5,34	$3\cdot10^2$
4	29,9	16,9	700	3,60	23,0	$5\cdot10^5$

$$\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\omega)=0,\quad \frac{\partial(\rho u)}{\partial t}+\operatorname{div}(\rho u\omega)+\frac{\partial P}{\partial z}=0,$$
$$\frac{\partial(\rho v)}{\partial t}+\operatorname{div}(\rho v\omega)+\frac{\partial P}{\partial r}=0, \tag{1}$$
$$\frac{\partial(\rho E)}{\partial t}+\operatorname{div}(\rho E\omega)+\operatorname{div}(P\omega)=0.$$

Initial and boundary conditions are given based on a cone for the cases of a plane shock wave with constant parameters behind the front and for a plane piston, moving according to a given law toward the cone vertex:

$$u(z,\ r,\ t)=u_s,\quad P(z,\ r,\ t)=P_s,\quad \rho(z,\ r,\ t)=\rho_s,$$
$$v(z,\ r,\ t)=0,\quad t\geqslant 0,\quad z=0,\quad 0\leqslant r\leqslant R; \tag{2}$$
$$u(z,\ r,\ t)=U(t),\quad 0\leqslant z\leqslant Ut,\quad 0\leqslant r\leqslant Ut\operatorname{tg}\alpha,\quad t\geqslant 0, \tag{3}$$

and in the volume of the cone:

$$u(z,\ r,\ t)=0,\quad v(z,\ r,\ t)=0,\quad P(z,\ r,\ t)=P_0,\quad \rho(z,\ r,\ t)=\rho_0. \tag{4}$$

The system of equations (1) with boundary conditions (2)-(4) was approximated by a finite-difference scheme of first order of accuracy for the method of large particles [14], modified to take into account the curvilinear and moving boundaries on an Euler grid [15]. We assume that there is no flow on the surface of the piston, walls of the cone, and axis of symmetry. The cone vertex is assumed to be blunt, and either a plane or an inscribed circle with radius $\delta \ll R$.

In order to test the calculation method, we obtain a numerical solution of the problem of the diffraction of a plane shock wave moving toward the vertex of a cone with dimensions L = 43.4 cm, R = 6.9 cm, and $\alpha = 10°$. The gas being investigated is argon, with initial parameters P_0 = 200 Pa and ρ_0 = 3.521 μg/cm³. On the boundary, at the base of the cone, the boundary conditions P_s, ρ_s, and u_s corresponded to the front of a shock wave with Mach numbers M = 6 and M = 10.2. The equation of state for argon was taken to be the ideal-gas equation $P = (\gamma - 1)\rho\varepsilon$ with adiabatic exponent $\gamma = 5/3$. Using a BÉSM-6 computer, the computation time was about 30 min for a single variant on a grid of 51 × 49 cells along the radius and axis of symmetry.

The time development of the two-dimensional structure of the gasdynamic flow is shown in Fig. 1. The distribution of the density isolines at the times: a) 31.51; b) 48.02; c) 62.06; d) 78.03; e) 90.02 μsec for M = 10.2 inside the cone was constructed from the maximum value ρ_m = 31.8 μg/cm³ (line 1) with step $\Delta\rho$ = 0.912 μg/cm³ and was numbered in the order of decreasing density. The position of the bow and reflected shock waves is determined by the bunching of isolines in the bow flow and along the maximum density gradient (dashed lines). As can be seen in Fig. 1a, the incoming shock wave is first reflected in Mach form at the wall of the cone, with the point of intersection of the three-wave structure being shifted ahead and in the direction of the symmetry axis. Then, after being reflected from

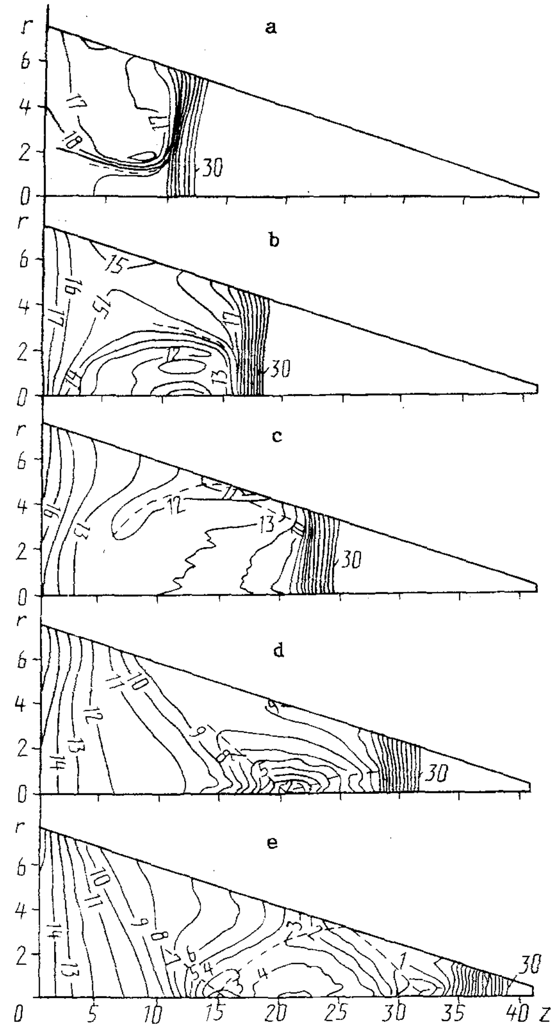

Fig. 1. Two-dimensional structure of gasdynamic flow inside a cone after an incoming shock wave (density isolines). z, r, cm.

the axis, it is shifted upward (Fig. 1b) and repeates these reflections twice (Fig. 1c-e). Following a reorganization of the bow structure there is a displacement and successive reflection of a "suspended" shock wave (dashed lines) on the side wall (Fig. 1c) and symmetry axis (Fig. 1d) — a double reflection on the axis and a single reflection on the wall (Fig. 1e). As can be seen from Fig. 2a and b, in accordance with the scheme (Fig. 1a), Mach reflection of the front is repeated on the walls and symmetry axis of the cone with a frequency that is always increasing and leads to a discontinuous increase and subsequent smooth decrease in shock-wave velocity. In Fig. 2b and c, we show a change in the velocity of the front D_S, calculated by the CCW method, for the condition of approach of the front to the axis of symmetry to the final minimum distance $\Delta r = 0.034$ mm (solid lines) and 0.0048 mm (dot—dash lines). Numerical values (small circles) of the shock-wave velocity are in satisfactory agreement with both theoretical and experimental data (triangles) [2] for both variants M = 10.2 (b) and M = 6 (c). The results given in Fig. 2c also show that in the ideal-gas approximation, the amplitude of a two-dimensional shock wave increases without bound for an asymptotic approximation both to the cone vertex and also to the axis of symmetry.* In this case, on the average, the velocity of the front along the z-axis increases approximately according to a power law (straight, inclined lines, Fig. 2c).

*A similar solution by the CCW method for the growth of the shock-waves amplitude with a decrease in cell dimensions in the neighborhood of focusing was obtained in one-dimensional calculations [16] by the finite-difference method [17].

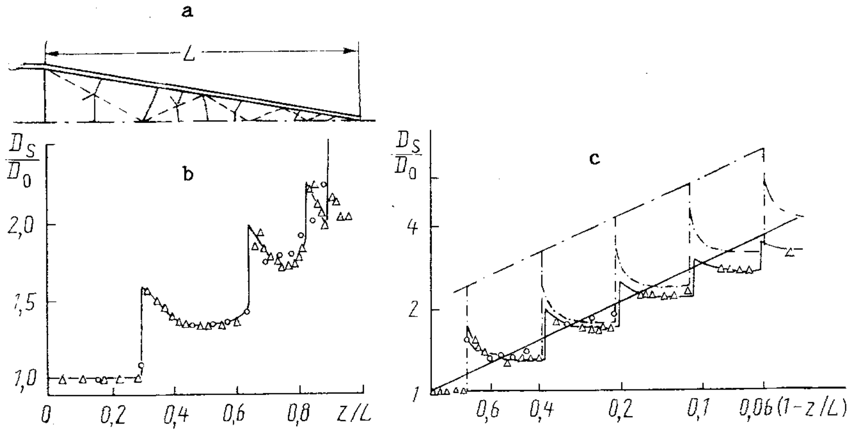

Fig. 2. Dependence of relative velocity of front D_S/D_0 on relative distance to vertex of cone z/L.

2. We consider the solution of the problem of the accumulation of shock waves generated by a plane piston accelerated under the action of laser radiation. Two-dimensional calculations for a more detailed solution on the axis of symmetry were not made because they require greater computer time. To compare the calculated data with experimental data [8, 9, 12], we chose cone dimensions R = 1 mm, $\alpha = 15°$, $\delta = 20$ μm; inside, we assumed deuterium with $P_0 = 50$ kPa, $\rho_0 = 0.09$ mg/cm^3. The actual thermodynamic properties of deuterium are given in Table 1, obtained from calculations based on the Saha model [18] with the adjustments proposed in [19]. The law of motion of a shell compressing deuterium was given in the form

$$U(t) = -a\ln(1 - t/t_1), \quad 0 \leqslant t \leqslant t_m < t_1.$$

According to [12], this expression is approximated by the "rocket model" of acceleration of a shell owing to recoil momentum for its evaporation; the constants a, t_m, and t_1 are determined experimentally from the laser pulse length and the measured velocity of the shell. In the calculation we assumed two piston acceleration regimes with attainment of maximum velocity $U(t_m) = 97$ km/sec (for $a = 13.3$ km/sec, $t_m = 44.5$ nsec, $t_1 = 44.53$ nsec) and 202 km/sec ($a = 26.6$ km/sec, $t_m = 20.02$ nsec, $t_1 = 20.03$ nsec). The first regime corresponded to a "heavy" shell made of polyethylene terephthalate of thickness 3 μm; the second regime used a lighter shell (1 μm) in the experiments [12]. The problem was solved, neglecting energy dissipation owing to electronic thermal conductivity, radiation transport, compressibility of the cone walls, which should ensure obtaining the upper boundary based on neutron yield. The number of neutrons, measured experimentally, was the single quantity based on which we could estimate the compression parameters. In the calculation, the number of neutrons was determined from the formula [20]

$$N = \int_0^t \int_V \langle \delta v \rangle \rho^2/2m_D^2 dV dt,$$

where $\langle\sigma v\rangle = 1{,}5\cdot10^{-9}T^{-2/3}\exp(-4250T^{-1/3})$, $m_D = 3{,}35\cdot10^{-24}$ g. For the motion of the piston toward the vertex of the cone we assumed: 1) the mass of the piston decreases in proportion to the instantaneous cross-sectional area of the cone, cut by rigid walls whose mass is neglected; 2) for $t > t_m$, acceleration of the piston by laser radiation stops, and it moves based on its inertia, slowing down only because of the counterpressure of the compressed deuterium.

For the calculated variants of compression of the gas by the piston, in the table we represent the gasdynamic quantities, the total energy of the plasma, and the total neutron yield when the values at the cone vertex are a maximum. The first three variants refer to a low-velocity acceleration regime, and are distinguished by the magnitude of the initial piston mass: $M_0 = 0.26$, 13, and 130 μg. The gas velocity during compression changes in jumps corresponding to reflections of a triple Mach configuration of shock waves on the axis of symmetry, and near the cone vertex is about 80 km/sec (variant 1) and 150-160 km/sec (variants 2 and 3). As can be seen from Table 1, neutron yield is not significant for the single reflection of shock waves. However, the motion of the "heavy" piston (variants 2

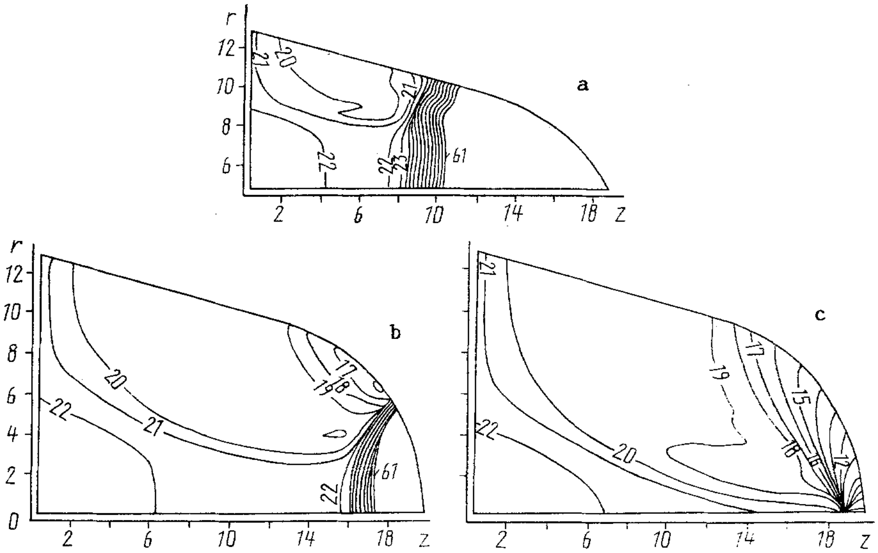

Fig. 3. Two-dimensional structure of gasdynamic flow inside a cone coupled with a spherical segment after entry of shock waves (pressure isolines). z, r, μm.

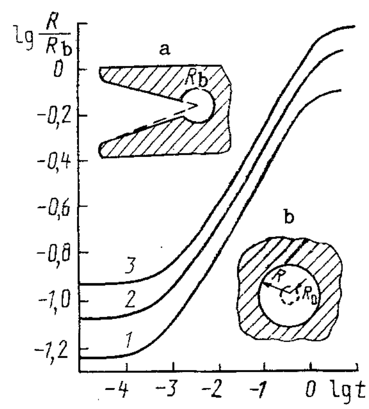

Fig. 4. Diagram of deformation of conical cavity in experiment (a), in the one-dimensional model formulation (b), and the calculated dependence of the radius of the cavity relative to its experimental value of R/R_b (1, 2, 3) on time t (μsec).

and 3) continues until it comes to a complete stop after the nonsingle reflection of shock waves from the vertex of the cone and the surface of the piston. In the calculation of the high-velocity regime (M_0 = 4.4 μg, variant 4) there is only a single neutron pulse, which ensures that, in spite of the strong damping of the piston after its maximum velocity is attained, the cumulative acceleration of the shock waves enables us to reach $D_S > 300$ km/sec. For compression of the "heavy" piston, two neutron pulses are observed, where the second ($N \sim 10^6$-10^7) is determined by the compression of deuterium by the piston in a quasi-isentropic regime. For both acceleration regimes, the number of neutrons obtained is an upper estimate, and exceeds by 1-2 orders of magnitude the experimental values N from [12].

3. The gasdynamic approximation, neglecting dissipative processes, in the solution of the problem of converging shock waves determines the unbounded increase of velocity, pressure, and temperature for the asymptotic approach of the shock-wave front to the cone

vertex. As we vary the gasdynamic parameters of damping of the flow, we consider if the plane vertex of the cone can be replaced by a hemisphere of radius δ = 10 μm inscribed in the cone (α = 15°).* In view of the limited computer possibilities, it is not possible to consider directly the complete process of evolution of Mach configurations from the acceleration of the piston to the diffraction of shock waves in spherical rounding. Therefore, we limit ourselves to a consideration of the final stage of flow, on which the gas, having parameters taken from the calculation of variant 4, flows in the region linking the cone and the spherical segment (a total length of about 20 μm along the z-axis). Calculations were carried out on a nonregular grid with 1300 cells and a division into 51 × 49 steps along r and z. On the basis of the conical part along the radius we assumed constant boundary conditions, ensuring for $t \geq 0$ ρ deuterium with parameters P_s 9.63 GPa, P_s = 0.415 mg/cm³, us = 300 km/sec, $\varepsilon_s = 3.53 \cdot 10^4$ kJ/g, corresponding to the data of variant 4.

The structure of the formulated gasdynamic flow is shown in Fig. 3 using an isobar constructed on a logarithmic scale from the maximum value 1.26 TPa (line 1) with step log ΔP = 0.1 at times a) 0.02, b) 0.04, and c) 0.047 nsec. Unlike the cases considered earlier (Fig. 1), the Mach wave formed at the entrance to the cone moves into the spherical segment without reflection at the axis of symmetry and is focused at the vertex of the spherical segment. The maximum values of the gasdynamic quantities are attained at the wall: P_m = 1.4 TPa, ρ_m = 2 mg/cm³, $T_m = 8.24 \cdot 10^7$ K, where the pressure at first is smoothly reduced with distance from the wall to ~1 μm; then it undergoes a sharp jump (bunching of the isobars in Fig. 3c). Then, this region is broadened in the opposite direction with velocity above 650 km/sec, forming behind the front of the reflected shock wave a section with comparatively uniform parameters: $T = 4.64 \cdot 10^7$ K, P = 0.3 TPa for t = 0.052 nsec. The results obtained indicate that in the gasdynamic formulation, the maximum parameters of compression of deuterium increase with account of sufficiently fine details of geometry in the focusing region and, evidently, for an increase in this region of the detail of the calculation grid.

Thus, for collision of the cumulative shock waves at the vertex of the frustum of the cone with radius δ = 20 μm the maximum values of the gasdynamic quantities are, in their own way, transient values and change with decreasing δ. In view of this, we should estimate how much these "transient" values correspond to real values. To do this, we consider the expansion of a spherical cavity of radius R_0 ($R_0 \approx R_b$, R_b is the radius of bulging of the vertex of the cone, observed in experiments [8, 12]), filled with deuterium having initial values of gasdynamic quantities, equal to the maximum P_m, ρ_m, and T_m in a medium mode of lead (Fig. 4). In this formulation, the one-dimensional problem of expansion of deuterium was solved using a finite-difference Lagrangian scheme with pseudoviscosity [17] with account of elastoplastic deformations [21] and real thermodynamic properties of lead, given by the tabular, wide-range equation of state [22, 23].

As the calculations showed, there is satisfactory agreement between the radius of the expanding cavity in lead with its experimental value R_b = 40 μm for the maximum parameters of filling, taken from the calculation of high-velocity compression (variant 4). This is seen from Fig. 4, where curves 1, 2, and 3 correspond to the initial values R_0 = 10, 15, and 20 μm for the same parameters of compressed deuterium.

The characteristic time of the considerable increase of volume of the cavity was ~10 nsec, comparable in value with the time for precompression of deuterium from a heavy shell in the quasi-isentropic regime (variants 2 and 3). Hence, in calculations of compression of a gas by a heavy, slow target, in the final stage we took account of the compressibility of the cone walls, the compliance of which can considerably vary the paameters of compression and neutron yield. In view of what has been said, and also taking into account the weak dependence of neutron yield $N \sim 10^4$-10^5 on the thickness of the shell and the angle α = 15-30° in experiments [12], we can conclude that heating and compression of deuterium with the appearance of neutrons can be carried out with the convergence of shock waves not only in a quasi-spherical, one-dimensional geometry, but also in a two-dimensional geometry with a plane wave front for only a single reflection of shock waves at the vertex of a conical target.

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*Such a rounding off of the radius of 10-20 μm holds in the preparation of conical targets for real experiments.

NOTATION

r, z, radial and axial coordinates; t, time; P, ρ, pressure and density; T, temperature; V, volume; E, total specific energy; ω, mass-velocity vector of gas; v, u, radial and axial components of gas velocity; Ω, solid angle of cone; R, radius of cone base; L, cone height; α, cone vertex angle; U(t), velocity of motion of shell-piston; δ, radius of curvature at cone vertex; $\varepsilon = E - \omega^2/2$, specific internal energy; γ, adiabatic exponent; M, Mach number; D, velocity of shock wave; a, t_1, t_m, constants in the law of motion of the shell-piston; m_D, mass of deuterion; N, number of neutrons; M_0, mass of piston; R_b, radius of bulging at the vertex of the cone, observed experimentally. Subscripts: 0, initial value; s, boundary conditions; m, maximum value.

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